

SCALE-UP: An implementation of Second-Principles DFT

Pablo García-Fernández, Jorge Íñiguez and
Javier Junquera

Louvain May 2019



Universidad de Cantabria



Collaborators

Cantabria University



Javier Junquera

Luxembourg Institute of Science and Technology



Jorge Iñiguez

Cantabria University Campus



Funding



RyC programme

What can we do with first-principles simulations?

Predict material properties using just fundamental constants

Little input from user necessary to obtain reliable information

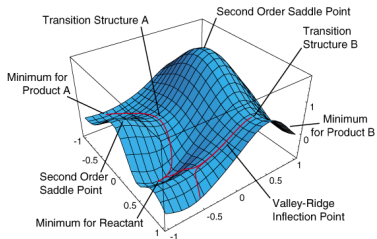
What can we do with first-principles simulations?

Predict material properties using just fundamental constants

Little input from user necessary to obtain reliable information

Solving the stationary Schrödinger equation (clamped nuclei):

$$\hat{H}\Psi = E\Psi \quad \text{or DFT}$$



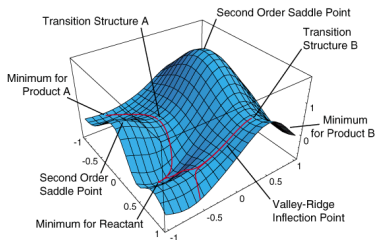
What can we do with first-principles simulations?

Predict material properties using just fundamental constants

Little input from user necessary to obtain reliable information

Solving the stationary Schrödinger equation (clamped nuclei):

$$\hat{H}\Psi = E\Psi \quad \text{or DFT}$$



- ✓ Predictive
- ✓ Accurate energies
- ✓ Equilibrium geometries

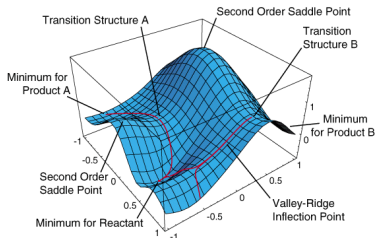
What can we do with first-principles simulations?

Predict material properties using just fundamental constants

Little input from user necessary to obtain reliable information

Solving the stationary Schrödinger equation (clamped nuclei):

$$\hat{H}\Psi = E\Psi \quad \text{or DFT}$$



- ✓ Predictive
- ✓ Accurate energies
- ✓ Equilibrium geometries
- ~ Excited states
- ~ Temperature
- ~ Defects

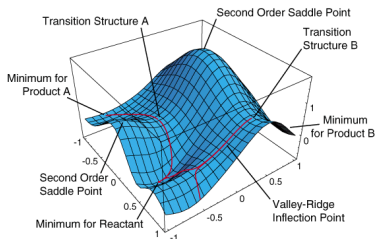
What can we do with first-principles simulations?

Predict material properties using just fundamental constants

Little input from user necessary to obtain reliable information

Solving the stationary Schrödinger equation (clamped nuclei):

$$\hat{H}\Psi = E\Psi \quad \text{or DFT}$$



- ✓ Predictive
- ✓ Accurate energies
- ✓ Equilibrium geometries
- ~ Excited states
- ~ Temperature
- ~ Defects
- ✗ Scaling (typically N^3)

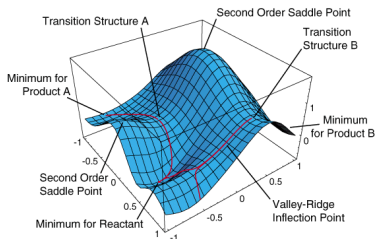
What can we do with first-principles simulations?

Predict material properties using just fundamental constants

Little input from user necessary to obtain reliable information

Solving the stationary Schrödinger equation (clamped nuclei):

$$\hat{H}\Psi = E\Psi \quad \text{or DFT}$$



- ✓ Predictive
- ✓ Accurate energies
- ✓ Equilibrium geometries
- ~ Excited states
- ~ Temperature
- ~ Defects
- ✗ Scaling (typically N^3)

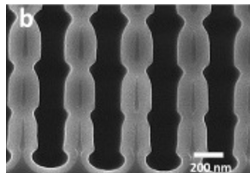
Wealth of information that could be difficult to obtain experimentally

The problem

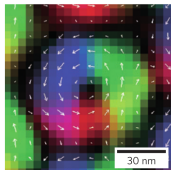
Scale of interest

✓ Nanoscale ($\approx 10 - 100\text{nm}$)

✗ DFT $\approx 1\text{ nm}$



Nanowires



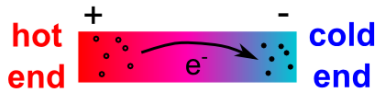
skyrmion in MnSi

The problem

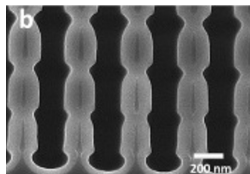
Scale of interest

✓ Nanoscale ($\approx 10 - 100\text{nm}$)

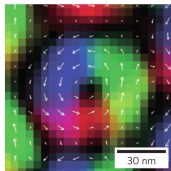
✗ DFT $\approx 1\text{ nm}$



Thermoelectrics, polarons...



Nanowires



skyrmion in MnSi

Non-equilibrium states

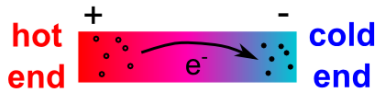
- ✓ Resistivity
- ✓ Charge diffusion
- ✓ Reaction yield

The problem

Scale of interest

✓ Nanoscale ($\approx 10 - 100\text{nm}$)

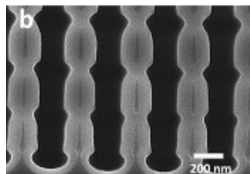
✗ DFT $\approx 1\text{ nm}$



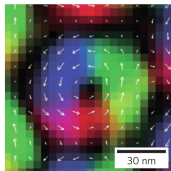
Thermoelectrics, polarons...

Disorder

- ✓ Domains
- ✓ Thermal
- ✓ Defects (polarons, impurities...)



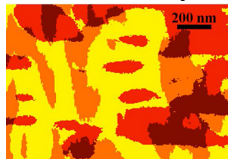
Nanowires



skyrmion in MnSi

Non-equilibrium states

- ✓ Resistivity
- ✓ Charge diffusion
- ✓ Reaction yield



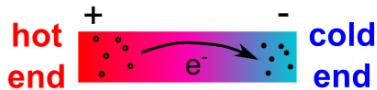
Ferroelectric domains in BiFeO₃

The problem

Scale of interest

✓ Nanoscale ($\approx 10 - 100\text{nm}$)

✗ DFT $\approx 1\text{ nm}$

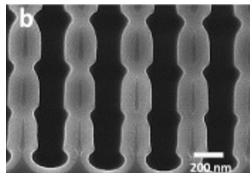


Thermoelectrics, polarons...

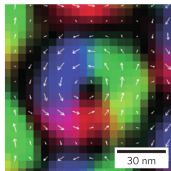
Disorder

- ✓ Domains
- ✓ Thermal
- ✓ Defects (polarons, impurities...)

- ▶ Perturbations/disorder are key elements in experiments.
- ▶ Room temperature is fundamental for applications.



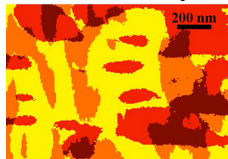
Nanowires



skyrmion in MnSi

Non-equilibrium states

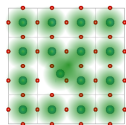
- ✓ Resistivity
- ✓ Charge diffusion
- ✓ Reaction yield



Ferroelectric domains in BiFeO₃

Are larger/faster FP simulations possible?

First principles simulations deal with all electrons in the system:



Number of electrons
grows fast

Hamiltonian $\sim N^2$

Diagonalization $\sim N^3$

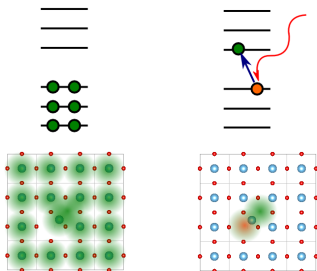
Are larger/faster FP simulations possible?

First principles simulations deal with all electrons in the system:

Number of electrons
grows fast

Hamiltonian $\sim N^2$

Diagonalization $\sim N^3$



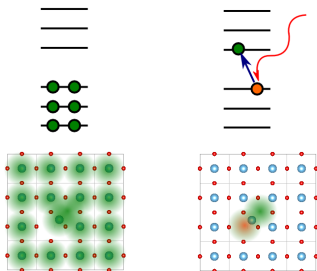
Response to
perturbations
usually involves a
few active
electron/holes

Are larger/faster FP simulations possible?

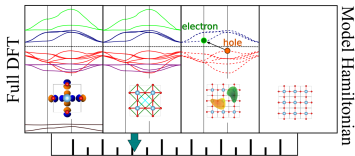
First principles simulations deal with all electrons in the system:

Number of electrons
grows fast

Hamiltonian $\sim N^2$
Diagonalization $\sim N^3$



Response to
perturbations
usually involves a
few active
electron/holes



Can we select the level of fidelity of
our calculations?

Can we make it efficient?

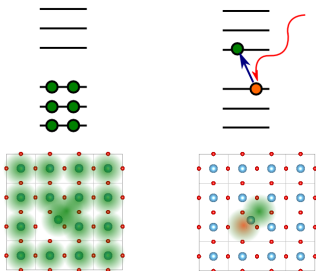
Can we reliably parameterize it?

Are larger/faster FP simulations possible?

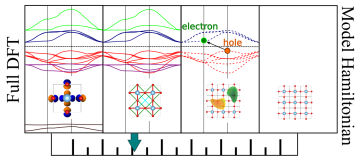
First principles simulations deal with all electrons in the system:

Number of electrons
grows fast

Hamiltonian $\sim N^2$
Diagonalization $\sim N^3$



Response to
perturbations
usually involves a
few active
electron/holes



Can we select the level of fidelity of
our calculations?

Can we make it efficient?

Can we reliably parameterize it?

Second-principles Density Functional Methods

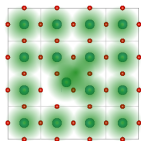
Basic concepts

We want to separate the active electrons that participate in physical properties from all others.

Basic concepts

We want to separate the active electrons that participate in physical properties from all others.

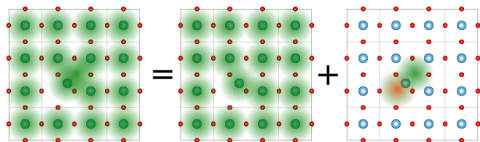
Suppose an insulator doped with electrons or holes:



Basic concepts

We want to separate the active electrons that participate in physical properties from all others.

Suppose an insulator doped with electrons or holes:



The total density is separated in **reference** and **deformation** densities:

$$n(\vec{r}) = n_0(\vec{r}) + \delta n(\vec{r})$$

n_0 = reference density

δn = deformation density

Approximating the DFT energy

Our starting point is the DFT energy

$$E_{\text{DFT}} = \sum_{j\vec{k}} o_{j\vec{k}} \langle \psi_{j\vec{k}} | \hat{t} + v_{\text{ext}} | \psi_{j\vec{k}} \rangle + \frac{1}{2} \iint \frac{n(\vec{r})n'(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3r d^3r' + E_{\text{xc}}[n] + E_{\text{nn}}$$

We want to write the energy in terms of the reference and deformation densities.

$$n(\vec{r}) = n_0(\vec{r}) + \delta n(\vec{r})$$

Approximating the DFT energy

Our starting point is the DFT energy

$$E_{\text{DFT}} = \sum_{j\vec{k}} o_{j\vec{k}} \langle \psi_{j\vec{k}} | \hat{t} + v_{\text{ext}} | \psi_{j\vec{k}} \rangle + \frac{1}{2} \iint \frac{n(\vec{r})n'(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3r d^3r' + E_{\text{xc}}[n] + E_{\text{nn}}$$

We want to write the energy in terms of the reference and deformation densities.

$$n(\vec{r}) = n_0(\vec{r}) + \delta n(\vec{r})$$

The only difficulty is the exchange-correlation energy that we expand in terms of δn (see e. g. M. Elstner et al., *Phys. Rev. B*, 58, 7260 (1998)):

$$E_{\text{xc}}[n] = E_{\text{xc}}[n_0] + \int \left. \frac{\delta E_{\text{xc}}}{\delta n(\vec{r})} \right|_{n_0} \delta n(\vec{r}) d^3r + \frac{1}{2} \iint \left. \frac{\delta^2 E_{\text{xc}}}{\delta n(\vec{r}) \delta n(\vec{r}')} \right|_{n_0} \delta n(\vec{r}) \delta n(\vec{r}') d^3r d^3r' + \dots$$

Approximating the DFT energy

Our starting point is the DFT energy

$$E_{\text{DFT}} = \sum_{j\vec{k}} o_{j\vec{k}} \langle \psi_{j\vec{k}} | \hat{t} + v_{\text{ext}} | \psi_{j\vec{k}} \rangle + \frac{1}{2} \iint \frac{n(\vec{r})n'(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3r d^3r' + E_{\text{xc}}[n] + E_{\text{nn}}$$

We want to write the energy in terms of the reference and deformation densities.

$$n(\vec{r}) = n_0(\vec{r}) + \delta n(\vec{r})$$

The only difficulty is the exchange-correlation energy that we expand in terms of δn (see e. g. M. Elstner et al., *Phys. Rev. B*, 58, 7260 (1998)):

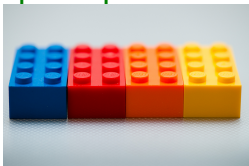
$$E_{\text{xc}}[n] = E_{\text{xc}}[n_0] + \int \left. \frac{\delta E_{\text{xc}}}{\delta n(\vec{r})} \right|_{n_0} \delta n(\vec{r}) d^3r + \frac{1}{2} \iint \left. \frac{\delta^2 E_{\text{xc}}}{\delta n(\vec{r}) \delta n(\vec{r}')} \right|_{n_0} \delta n(\vec{r}) \delta n(\vec{r}') d^3r d^3r' + \dots$$

As in usual TB-DFT approximations, we cut at second-order

$$E_{\text{DFT}} \approx E = E^{(0)} + E^{(1)} + E^{(2)}$$

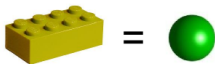
However, we group the terms in a different way to TB-DFT.

Second-principles DFT approach



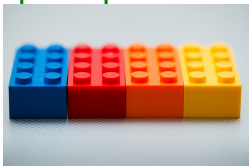
Material simulations
allow for various approaches

First principles methods are **atomistic** with **flexible detailed bonding**
FP or TB-DFT



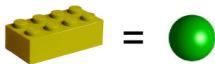
Based on atoms

Second-principles DFT approach



Material simulations
allow for various approaches

First principles methods are **atomistic** with **flexible detailed bonding**
FP or TB-DFT

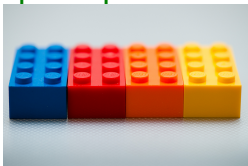


Based on atoms

$$E_{\text{DFT}} \approx \underbrace{E^{(0)}}_{\text{atomic cores}} + \underbrace{E^{(1)}}_{\text{full 1e energy}} + \underbrace{E^{(2)}}_{\text{full 2e energy}} + \dots$$

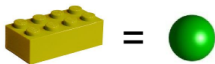
Atoms \implies FP \implies Materials

Second-principles DFT approach



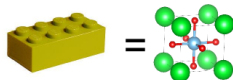
Material simulations
allow for various approaches

First principles methods are **atomistic** with **flexible detailed bonding**
FP or TB-DFT



Based on atoms

$$E_{\text{DFT}} \approx \underbrace{E^{(0)}}_{\text{atomic cores}} + \underbrace{E^{(1)}}_{\text{full 1e energy}} + \underbrace{E^{(2)}}_{\text{full 2e energy}} + \dots$$

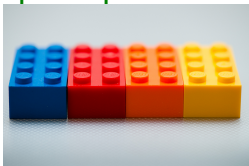


Based on materials

$$E_{\text{DFT}} \approx \underbrace{E^{(0)}}_{\text{lattice}} + \underbrace{E^{(1)} + E^{(2)} + \dots}_{\text{electron excitations}}$$

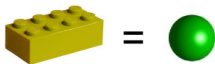
Atoms \implies FP \implies Materials \implies SP \implies Large-scale
Accurate properties do not require bond-breaking!

Second-principles DFT approach



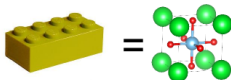
Material simulations
allow for various approaches

First principles methods are **atomistic** with **flexible detailed bonding**
FP or TB-DFT



Based on atoms

$$E_{\text{DFT}} \approx \underbrace{E^{(0)}}_{\text{atomic cores}} + \underbrace{E^{(1)}}_{\text{full 1e energy}} + \underbrace{E^{(2)}}_{\text{full 2e energy}} + \dots$$



Based on materials

$$E_{\text{DFT}} \approx \underbrace{E^{(0)}}_{\text{lattice}} + \underbrace{E^{(1)} + E^{(2)} + \dots}_{\text{electron excitations}}$$

Atoms \implies FP \implies Materials \implies SP \implies Large-scale
Accurate properties do not require bond-breaking!

Precise, small, material-adapted basis \rightarrow Wannier-like functions

I. Souza et al., *Phys. Rev. B*, 65, 035109 (2001)

Energy terms: $E = E^{(0)} + E^{(1)} + E^{(2)}$ Reference

This term is the full DFT energy for the reference state:

$$E^{(0)} = \sum_{j\vec{k}} o_{j\vec{k}}^0 \langle \psi_{j\vec{k}}^0 | \hat{t} + v_{\text{ext}} | \psi_{j\vec{k}}^0 \rangle + \frac{1}{2} \iint \frac{n_0(\vec{r})n_0'(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3r d^3r' + E_{\text{xc}}[n_0] + E_{\text{nn}}$$

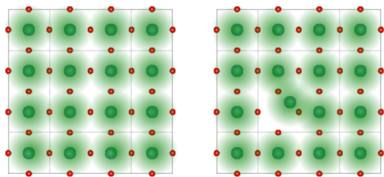
Energy terms: $E = E^{(0)} + E^{(1)} + E^{(2)}$ Reference

This term is the full DFT energy for the reference state:

$$E^{(0)} = \sum_{j\vec{k}} o_{j\vec{k}}^0 \langle \psi_{j\vec{k}}^0 | \hat{t} + v_{\text{ext}} | \psi_{j\vec{k}}^0 \rangle + \frac{1}{2} \iint \frac{n_0(\vec{r})n_0'(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3r d^3r' + E_{\text{xc}}[n_0] + E_{nn}$$

No approximations

At difference with usual TB-DFT this term is very large and contains most of the total energy. It can be made really accurate.



The reference state is defined for all geometries

$E_0(\eta, \{\vec{u}\})$ is the energy surface for the reference state

It can be represented by a high-quality model potential.

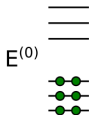
J. Wojdeł et al., *JPCM*, 25, 305401 (2013)

This lattice Hamiltonian is implemented in Multibinit!

Energy terms: $E = E^{(0)} + E^{(1)} + E^{(2)}$ One electron

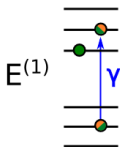
Reference

- ▶ Full DFT energy for n_0
- ▶ Force field
- ▶ **Multibinit**



$E^{(1)}$ contains differences in one-electron energies

$$E^{(1)} = \sum_{j\vec{k}} \left[o_{j\vec{k}} \langle \psi_{j\vec{k}} | \hat{h}_0 | \psi_{j\vec{k}} \rangle - o_{j\vec{k}}^0 \langle \psi_{j\vec{k}}^0 | \hat{h}_0 | \psi_{j\vec{k}}^0 \rangle \right]$$



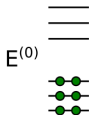
where \hat{h}_0 is the Kohn-Sham Hamiltonian for the reference density:

$$\hat{h}_0 = \hat{t} + v_{\text{ext}} + v_{\text{H}}(n_0) + v_{\text{xc}}[n_0]$$

Energy terms: $E = E^{(0)} + E^{(1)} + E^{(2)}$ One electron

Reference

- ▶ Full DFT energy for n_0
- ▶ Force field
- ▶ **Multibinit**



$E^{(1)}$ contains differences in one-electron energies

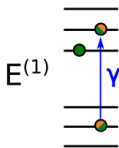
$$E^{(1)} = \sum_{j\vec{k}} \left[o_{j\vec{k}} \langle \psi_{j\vec{k}} | \hat{h}_0 | \psi_{j\vec{k}} \rangle - o_{j\vec{k}}^0 \langle \psi_{j\vec{k}}^0 | \hat{h}_0 | \psi_{j\vec{k}}^0 \rangle \right]$$

$$= \sum_{ab} D_{ab} \gamma_{ab} \quad (\text{Wannier basis, } \chi_a)$$

$$\delta n(\vec{r}) = \sum_{ab} D_{ab} \chi_a^*(\vec{r}) \chi_b(\vec{r})$$

γ_{ab} takes the role of the hopping constant in TB schemes.

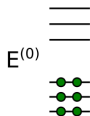
$$\gamma_{ab} = \int d^3r \chi_a^*(\vec{r}) \hat{h}_0 \chi_b(\vec{r})$$



Energy terms: $E = E^{(0)} + E^{(1)} + E^{(2)}$ One electron

Reference

- ▶ Full DFT energy for n_0
- ▶ Force field
- ▶ **Multibinit**



$E^{(1)}$ contains differences in one-electron energies

$$E^{(1)} = \sum_{j\vec{k}} \left[o_{j\vec{k}} \langle \psi_{j\vec{k}} | \hat{h}_0 | \psi_{j\vec{k}} \rangle - o_{j\vec{k}}^0 \langle \psi_{j\vec{k}}^0 | \hat{h}_0 | \psi_{j\vec{k}}^0 \rangle \right]$$

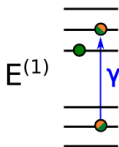
$$= \sum_{ab} D_{ab} \gamma_{ab} \quad (\text{Wannier basis, } \chi_a)$$

$$\delta n(\vec{r}) = \sum_{ab} D_{ab} \chi_a^*(\vec{r}) \chi_b(\vec{r})$$

γ_{ab} takes the role of the hopping constant in TB schemes.

$$\gamma_{ab} = \int d^3r \chi_a^*(\vec{r}) \hat{h}_0 \chi_b(\vec{r})$$

Only depends on difference density!



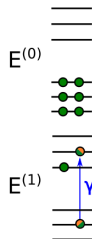
Energy terms: $E = E^{(0)} + E^{(1)} + E^{(2)}$ Two electron

Reference

- ▶ Full DFT energy for n_0
- ▶ Force field
- ▶ **Multibinit**

One-electron

- ▶ Depends only on difference density
- ▶ Tight-binding like



$E^{(2)}$ are interactions between 2 electrons ($E^{(3)}$ 3-electron, etc.):

$$E^{(2)} = \frac{1}{2} \int d^3r \int d^3r' g(\vec{r}, \vec{r}', s, s') \delta n(\vec{r}, s) \delta n(\vec{r}', s')$$

where g is a screened electron-electron interaction operator.

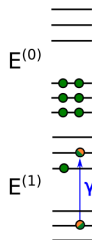
Energy terms: $E = E^{(0)} + E^{(1)} + E^{(2)}$ Two electron

Reference

- ▶ Full DFT energy for n_0
- ▶ Force field
- ▶ **Multibinit**

One-electron

- ▶ Depends only on difference density
- ▶ Tight-binding like



$E^{(2)}$ are interactions between 2 electrons ($E^{(3)}$ 3-electron, etc.):

$$E^{(2)} = \frac{1}{2} \int d^3r \int d^3r' g(\vec{r}, \vec{r}', s, s') \delta n(\vec{r}, s) \delta n(\vec{r}', s')$$

$$\left\{ \left[D_{ab}^\uparrow + D_{ab}^\downarrow \right] \left[D_{a'b'}^\uparrow + D_{a'b'}^\downarrow \right] U_{aba'b'} \right.$$

$$\left. + \left[D_{ab}^\uparrow - D_{ab}^\downarrow \right] \left[D_{a'b'}^\uparrow - D_{a'b'}^\downarrow \right] I_{aba'b'} \right\}$$

where g is a screened electron-electron interaction operator.

$E^{(2)}$ only depends the difference density

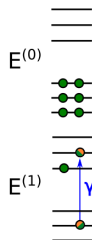
Energy terms: $E = E^{(0)} + E^{(1)} + E^{(2)}$ Two electron

Reference

- ▶ Full DFT energy for n_0
- ▶ Force field
- ▶ **Multibinit**

One-electron

- ▶ Depends only on difference density
- ▶ Tight-binding like



$E^{(2)}$ are interactions between 2 electrons ($E^{(3)}$ 3-electron, etc.):

$$E^{(2)} = \frac{1}{2} \int d^3r \int d^3r' g(\vec{r}, \vec{r}', s, s') \delta n(\vec{r}, s) \delta n(\vec{r}', s')$$

$$\left\{ \left[D_{ab}^\uparrow + D_{ab}^\downarrow \right] \left[D_{a'b'}^\uparrow + D_{a'b'}^\downarrow \right] U_{aba'b'} \right.$$

$$\left. + \left[D_{ab}^\uparrow - D_{ab}^\downarrow \right] \left[D_{a'b'}^\uparrow - D_{a'b'}^\downarrow \right] I_{aba'b'} \right\}$$

where g is a screened electron-electron interaction operator.

$E^{(2)}$ only depends the difference density

Energy terms: $E = E^{(0)} + E^{(1)} + E^{(2)}$ Calculation

Reference

- ▶ Full DFT energy for n_0
- ▶ Force field
- ▶ **Multibinit**

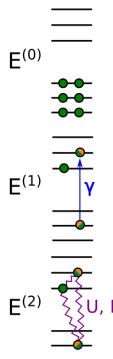
One-electron

- ▶ Depends only on difference density
- ▶ Tight-binding like

Two-electron

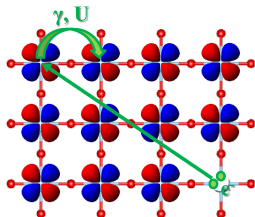
- ▶ Depends only on difference density
- ▶ Screened mean-field interactions

- ▶ **Accurate**
- ▶ **Fast**
- ▶ **Valid for all kind of systems (magnetic, metallic, ...)**



Electrostatics/Electron-lattice interactions

All interactions occur between localized objects:



At long-range (far-field regime) shape of source density is unimportant

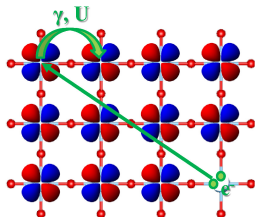
Multipolar expansion

γ and U contain electrostatic (long-range) contributions

Hartree/electron-nucleus

Electrostatics/Electron-lattice interactions

All interactions occur between localized objects:



At long-range (far-field regime) shape of source density is unimportant

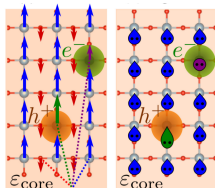
Multipolar expansion

γ and U contain electrostatic (long-range) contributions

Hartree/electron-nucleus

Local dipoles:

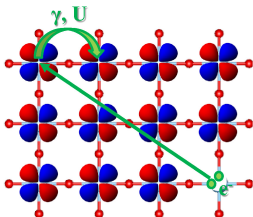
- ▶ Atomic displacement $\rightarrow Z^*$
- ▶ Hybridizations $\rightarrow \langle \chi_a | \vec{r} | \chi_b \rangle$



Potential approximated by field of point charges and dipoles localized at the reference geometry

Electrostatics/Electron-lattice interactions

All interactions occur between localized objects:



At long-range (far-field regime) shape of source density is unimportant

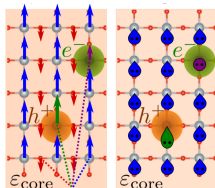
Multipolar expansion

γ and U contain electrostatic (long-range) contributions

Hartree/electron-nucleus

Local dipoles:

- ▶ Atomic displacement $\rightarrow Z^*$
- ▶ Hybridizations $\rightarrow \langle \chi_a | \vec{r} | \chi_b \rangle$

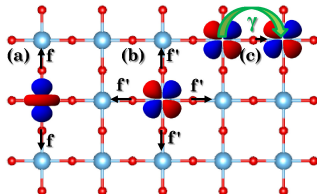


Potential approximated by field of point charges and dipoles localized at the reference geometry

Model parameters \rightarrow long and short range contributions.

Geometry dependence - Forces

Geometry is involved by expanding γ on the atomic positions:

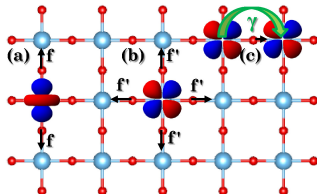


$$\gamma_{ab}^{sr} = \gamma_{ab}^0 + \sum_{\lambda\nu} \left[\vec{f}_{ab,\lambda\nu} \cdot \delta\vec{r}_{\lambda\nu} + \delta\vec{r}_{\lambda\nu} \cdot \overset{\leftrightarrow}{g}_{ab,\lambda\nu} \cdot \delta\vec{r}_{\lambda\nu} + \dots \right]$$

U, I should also depend on $\delta\vec{r}$ \rightarrow neglected

Geometry dependence - Forces

Geometry is involved by expanding γ on the atomic positions:



$$\gamma_{ab}^{sr} = \gamma_{ab}^0 + \sum_{\lambda\nu} \left[\vec{f}_{ab,\lambda\nu} \cdot \delta\vec{r}_{\lambda\nu} + \delta\vec{r}_{\lambda\nu} \cdot \overset{\leftrightarrow}{g}_{ab,\lambda\nu} \cdot \delta\vec{r}_{\lambda\nu} + \dots \right]$$

U,I should also depend on $\delta\vec{r} \rightarrow$ neglected

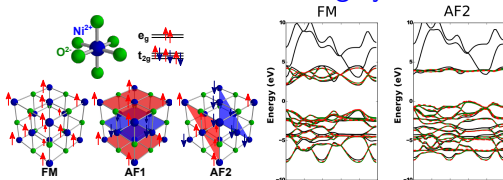
The forces are then obtained:

$$\vec{F}_{\lambda} = -\vec{\nabla}_{\lambda} E = -\vec{\nabla}_{\lambda} E^{(0)} - \sum_{ab} D_{ab} \vec{\nabla}_{\lambda} \gamma_{ab}$$

Electronic contribution corrects the force field

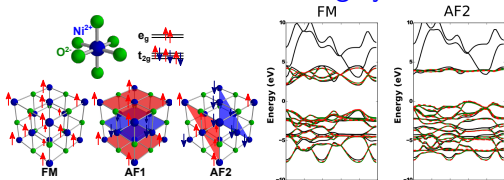
Applications of SPDF

NiO - Insulator with highly correlated electrons:



Applications of SPDF

NiO - Insulator with highly correlated electrons:



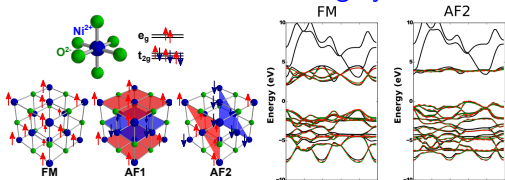
Magnetic Properties

| Method | J_1 (meV) | J_2 (meV) |
|-------------------|-------------|-------------|
| neutron | 1.4 | -19.0 |
| LDA+U | 2.6 | -17.5 |
| SP-Ni(3d) + O(2p) | 3.3 | -17.6 |

Electronic/magnetic properties predicted at DFT level
2000 atoms single point 1 hour in 1 Desktop cpu (full diag)

Applications of SPDF

NiO - Insulator with highly correlated electrons:

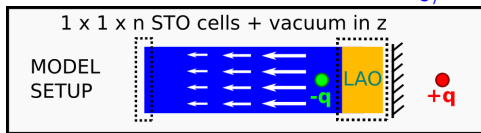


Magnetic Properties

| Method | J_1 (meV) | J_2 (meV) |
|-------------------|-------------|-------------|
| neutron | 1.4 | -19.0 |
| LDA+U | 2.6 | -17.5 |
| SP-Ni(3d) + O(2p) | 3.3 | -17.6 |

Electronic/magnetic properties predicted at DFT level
2000 atoms single point 1 hour in 1 Desktop cpu (full diag)

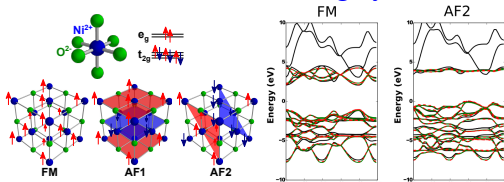
2DEG at SrTiO₃/LaAlO₃ Interface



FP → M. Stengel, *PRL*, 106, 136803 (2011)

Applications of SPDFT

NiO - Insulator with highly correlated electrons:

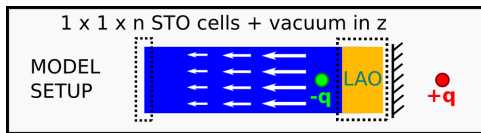


Magnetic Properties

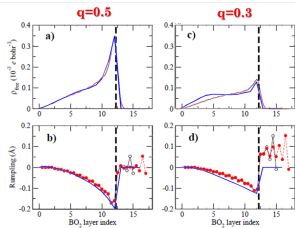
| Method | J_1 (meV) | J_2 (meV) |
|-------------------|-------------|-------------|
| neutron | 1.4 | -19.0 |
| LDA+U | 2.6 | -17.5 |
| SP-Ni(3d) + O(2p) | 3.3 | -17.6 |

Electronic/magnetic properties predicted at DFT level
2000 atoms single point 1 hour in 1 Desktop cpu (full diag)

2DEG at SrTiO₃/LaAlO₃ Interface



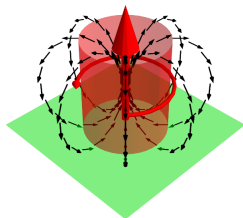
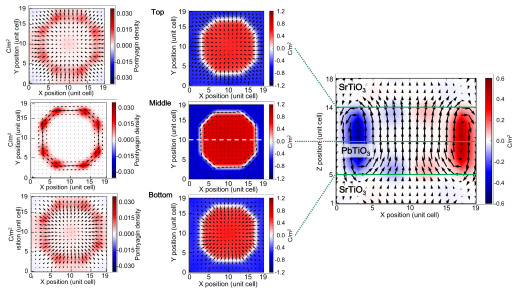
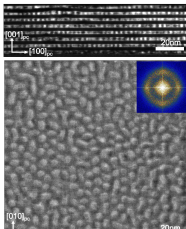
FP → M. Stengel, *PRL*, 106, 136803 (2011)



SPDFT captures doping and lattice screening!

Ferroelectric bubbles

SP can simulate ferroelectric bubble skyrmions in $\text{PbTiO}_3/\text{SrTiO}_3$



- ▶ ~ 40000 atoms
- ▶ Tangential polarization to bubble
Makes bubbles chiral
Explains XCD signal
- ▶ Bubble shows non-trivial topology
Planes \rightarrow same topological charge

S. Das et al., *Nature*, 568, 368 (2019)

The implementation of SP-DFT: SCALE-UP



P. Garcia-Fernandez, J. Wojdeł, J. Iñiguez and J. Junquera
Phys. Rev. B, **93**, 195137 (2016)

- ▶ Fully integrated electron+lattice models
- ▶ Single-points, Dynamics (isokinetic, Langevin), Montecarlo
- ▶ SCF with convergence acelerators
- ▶ TDDFT: Real-time density propagation, $\vec{E}(t)$ fields

The implementation of SP-DFT: SCALE-UP



P. Garcia-Fernandez, J. Wojdeł, J. Iñiguez and J. Junquera
Phys. Rev. B, **93**, 195137 (2016)

- ▶ Fully integrated electron+lattice models
- ▶ Single-points, Dynamics (isokinetic, Langevin), Montecarlo
- ▶ SCF with convergence acelerators
- ▶ TDDFT: Real-time density propagation, $\vec{E}(t)$ fields
- ▶ Fortran 90 code + python utilities and interface
- ▶ Parallelization: Hybrid scheme, MPI+OPENMP
- ▶ SCALE-UP python module: Running and analysis
- ▶ Model building suite: MODELMAKER

The implementation of SP-DFT: SCALE-UP

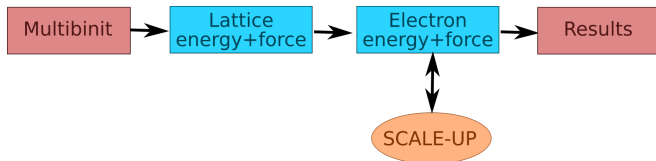


P. Garcia-Fernandez, J. Wojdeł, J. Iñiguez and J. Junquera
Phys. Rev. B, **93**, 195137 (2016)

- ▶ Fully integrated electron+lattice models
- ▶ Single-points, Dynamics (isokinetic, Langevin), Montecarlo
- ▶ SCF with convergence acelerators
- ▶ TDDFT: Real-time density propagation, $\vec{E}(t)$ fields
- ▶ Fortran 90 code + python utilities and interface
- ▶ Parallelization: Hybrid scheme, MPI+OPENMP
- ▶ SCALE-UP python module: Running and analysis
- ▶ Model building suite: MODELMAKER
- ▶ Future: Spin-orbit, defects (surfaces, interfaces, impurities...)
- ▶ Future: Pure density-matrix implementation

The connection between SCALE-UP and MULTIBINIT

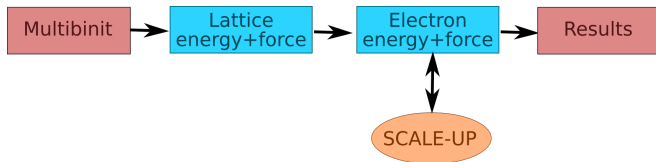
SCALE-UP can be run as a library by other codes



- ▶ It can provide $E^{(0)}$, $E^{(1)}+E^{(2)}$ or $E^{(0)}+E^{(1)}+E^{(2)}$

The connection between SCALE-UP and MULTIBINIT

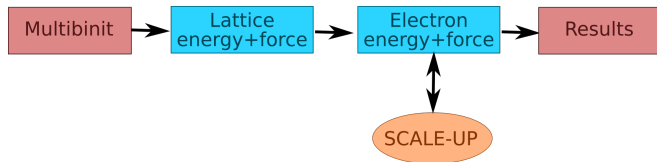
SCALE-UP can be run as a library by other codes



- ▶ It can provide $E^{(0)}$, $E^{(1)}+E^{(2)}$ or $E^{(0)}+E^{(1)}+E^{(2)}$
- ▶ It has three main calls
 - 1 `scaleup_init`
 - 2 `calculate_energy`
 - 3 `calculate_forces`

The connection between SCALE-UP and MULTIBINIT

SCALE-UP can be run as a library by other codes



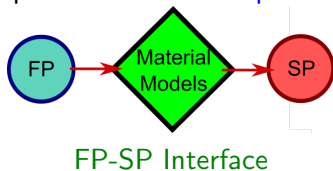
- ▶ It can provide $E^{(0)}$, $E^{(1)}+E^{(2)}$ or $E^{(0)}+E^{(1)}+E^{(2)}$
- ▶ It has three main calls
 - 1 `scaleup_init`
 - 2 `calculate_energy`
 - 3 `calculate_forces`

Distribution SCALE-UP → Attendance to workshop

Rationale: Forming community, second-principles are not blackbox

Model construction

Second Principles depend on First Principles to create models

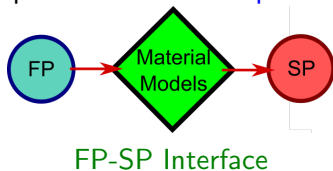


Very delicate process, we need:

- ▶ **Accuracy:** SP model needs to reproduce FP
- ▶ **Lightweight:** SP model needs to be efficient

Model construction

Second Principles depend on First Principles to create models

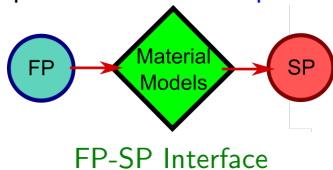


Very delicate process, we need:

- ▶ **Accuracy:** SP model needs to reproduce FP
- ▶ **Lightweight:** SP model needs to be efficient
- ▶ **Robust:** Necessary characteristics of model construction tool
 - 1 Automated

Model construction

Second Principles depend on First Principles to create models

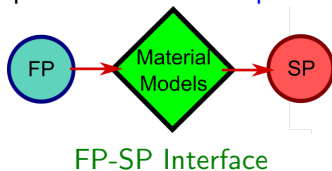


Very delicate process, we need:

- ▶ **Accuracy:** SP model needs to reproduce FP
- ▶ **Lightweight:** SP model needs to be efficient
- ▶ **Robust:** Necessary characteristics of model construction tool
 - 1 Automated
 - 2 Few and clean input parameters
 - 3 Systematically improvable models

Model construction

Second Principles depend on First Principles to create models



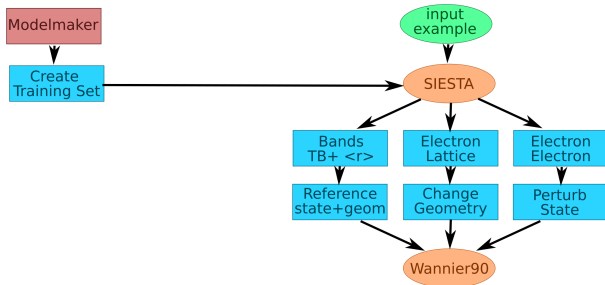
Very delicate process, we need:

- ▶ **Accuracy:** SP model needs to reproduce FP
- ▶ **Lightweight:** SP model needs to be efficient
- ▶ **Robust:** Necessary characteristics of model construction tool
 - 1 Automated
 - 2 Few and clean input parameters
 - 3 Systematically improvable models
 - 4 Produces hopping, electron-lattice, electron-electron-parameters

Currently we have a **SCALE-UP - SIESTA - WANNIER90 Interface**

Modelmaker

The first step to create a model is to create a FP **training set**



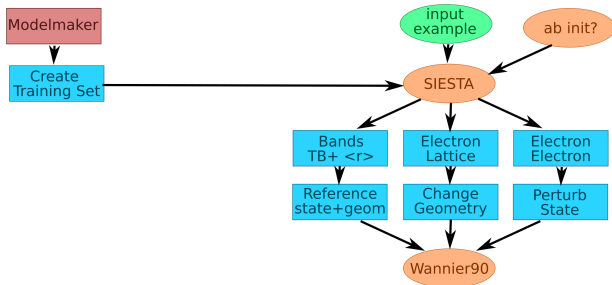
$$h_{ab}^s = \gamma_{ab}^{\text{RAG},sr} + \delta\gamma_{ab}^{\text{el-lat},sr}(\{\vec{u}_\lambda\}) + \sum_{a'b'} \left(D_{a'b'}^U U_{aba'b'} \pm D_{a'b'}^I I_{aba'b'} \right) + \gamma_{ab}^{\text{lr}}$$

Model Hamiltonian \rightarrow capture FP physics

- ▶ **Bands:** Wannier Hamiltonian
- ▶ **Electron-lattice terms:** Change geometry on a supercell
- ▶ **Electron-electron terms:** Controlled perturbation of electrons

Modelmaker

The first step to create a model is to create a FP **training set**



$$h_{ab}^s = \gamma_{ab}^{\text{RAG},sr} + \delta\gamma_{ab}^{\text{el-lat},sr}(\{\vec{u}_\lambda\}) + \sum_{a'b'} \left(D_{a'b'}^U U_{aba'b'} \pm D_{a'b'}^I I_{aba'b'} \right) + \gamma_{ab}^{\text{lr}}$$

Model Hamiltonian \rightarrow capture FP physics

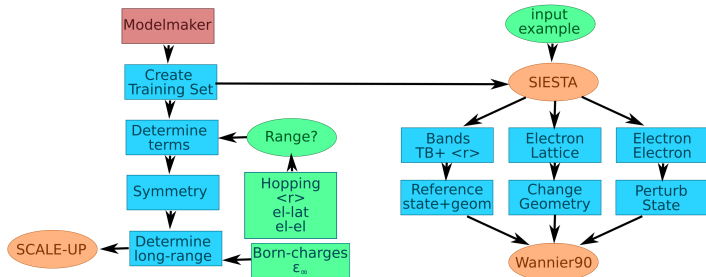
- ▶ **Bands:** Wannier Hamiltonian
- ▶ **Electron-lattice terms:** Change geometry on a supercell
- ▶ **Electron-electron terms:** Controlled perturbation of electrons

The SIESTA-WANNIER90 interface...

Javier Junquera

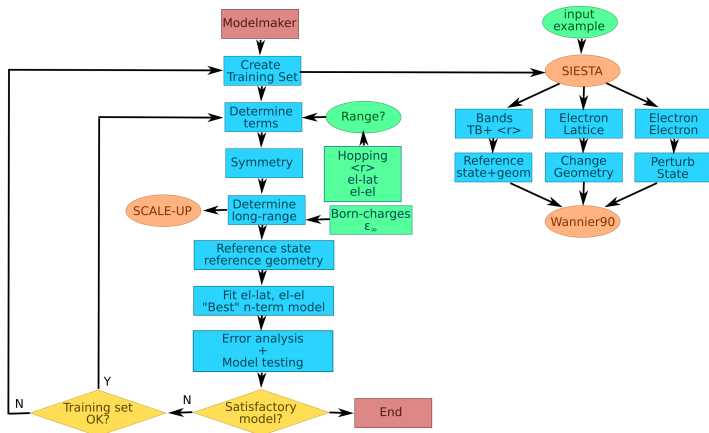
Modelmaker

Next we need to **determine the terms** that form the model



- ▶ The input is just 4 distances that determine the range of **hopping**, **position**, **electron-lattice** and **electron-electron** matrix elements.
- ▶ The code then filters the created terms using **symmetry**
- ▶ Finally, modelmaker calls SCALE-UP to determine the long-range corrections

Modelmaker



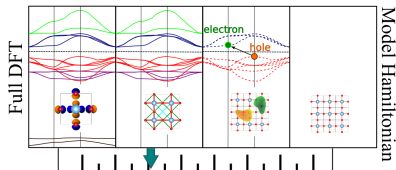
- ▶ The code determines iteratively "best n-terms" models
- ▶ It provides with tools to check accuracy of the model and systematic error detection
- ▶ The user decides if model is good enough and how to improve

Summary

- ▶ Second-Principles DFT bridges the gap between first-principles and model Hamiltonians
- ▶ The goal is getting closer to "Computational experiments"

P. Garcia-Fernandez et al., *PRB*, 93, 195137 (2016)

J. Wojdeł et al., *JPCM*, 25, 305401 (2013)



- ▶ Current applications show the accuracy and versatility of method
S. Das et al., *Nature*, 568, 368 (2019)
- ▶ Their use requires still a lot of user input
Strong interactions with FP codes

Thank you for your attention!